INTEGRATING CALCULUS AND GENERAL PHYSICS USING A WORKSHOP AND PEER-LEADER APPROACH

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An integrated introductory physics and calculus course is being developed at the University of Portland to address several persistent problems in student learning: poor conceptualization and retention of material, under-developed problem-solving skills, and difficulty actively applying knowledge across disciplines. The course and dedicated laboratory will also improve training of both pre-service and in-service K-12 teachers. The course will be integrated in terms of content, pedagogy and classroom design. Going beyond a "just in time" approach in which mathematics is often subordinate to physics, we take advantage of the integration, organizing the course around 'threads' in which the subjects reinforce and motivate each other. Instructors will have flexibility in the timing of topics, at times focusing on building strong foundations and at others, establishing connections between subjects. We will integrate and adapt several approaches that have proven successful (including workshop physics and peer-led team learning) and build on successful projects at UP using cooperative learning models. Working in small groups on investigative activities with supervising peer-leaders and a faculty member overseeing the entire class, students will make connections across disciplines in one classroom environment using a common set of tools. Lecture, hands-on learning, and computer usage will be integrated with group activities such as experimentation and joint problem-solving. Using the PLTL model, the conference workshop will follow one or more 'threads' through a semester. Participants will also have the opportunity to suggest, discuss, and develop additional activities supporting these threads.

Integrating introductory physics and calculus: overview

Problems in student learning such as poor conceptualization and retention of material, under-developed problem-solving skills, and difficulty actively applying knowledge across disciplines are widespread in calculus and physics education. Evidence of these difficulties in student learning is easy to find. Many researchers [1-6] have documented that introductory physics students enter with misconceptions that are not corrected by traditional instruction and that traditional assessment methods do not reveal the persistence of many fundamental errors. The disconnect between math and physics instruction has also received attention, for example in the work of Steinberg, Wittmann and Redish [7] and Beichner *et al.*[8] At the University of Portland the increasingly poor mathematical preparation of incoming students puts an added burden on physics instruction by requiring extensive review of mathematics (as documented at UP in a pre-test given to first semester introductory physics students from 1990 to 1999.) We have also seen that students at UP starting both calculus and physics at the same time fail to complete both courses (especially physics) at a much higher rate than students who start the physics sequence after having taken a semester of calculus. Many of our students in these courses are engineering majors, whose schedules make deferring physics for a semester difficult.

To address these issues, we are developing an integrated calculus and introductory physics course and classroom. The integration is realized in three ways: syllabus design, pedagogy, and classroom design. In terms of content, we take full advantage of the integration by organizing the course around 'threads' in which the subjects reinforce and motivate each other. This goes beyond a "just in time" approach in which mathematics is often subordinate to physics. By carefully timing the math and physics content we take advantage of the opportunities each subject has to motivate and reinforce the other. In term of class structure, students will work in small groups at stations designed to promote interaction within and among groups. Peer-leaders, each taking charge of three groups of students, will maintain and encourage student interest and focus on conceptual understanding through a Socratic questioning dialog. Finally, in terms of the classroom, the stations will incorporate a seemless interface with technology. The course will be taught in a dedicated classroom with workstations each serving one student group, and each with a computer networked to the other computers in the classroom. Mathematica will provide a powerful set of software tools available simultaneously for both mathematics and physics. Using experimental tools and hands-on activities, students will discover physical principles and model mathematical concepts, actively constructing their own knowledge. In physics, this common set of tools will support the use of recently acquired mathematical skills, while in mathematics the connection to physics will drive deeper conceptual understanding of the mathematical concepts.

We are adapting and coordinating several complementary approaches into a comprehensive program: Workshop Physics (WP) [9,10], Peer-Led Team Learning (PLTL) [11] and curricular materials for integrating calculus and physics developed at University of New

Hampshire (UNH) [12] . Using WP and UNH materials, students will learn through hands-on activities. Overlaying this will be the PLTL model in which a peer-leader questions, probes and guides groups of students.

Course structure and content

In our integrated course, the same students will be enrolled in a section of calculus and of physics, but will receive separate credit and grades for the two subjects. This course will be scheduled for five, two hour sessions per week. On average, two of these sessions will concentrate on calculus and three on physics, but we have the flexibility to spend a larger proportion of time on physics or on math as the material dictates.

CONTENT

The following scenarios are representative examples of how we will take advantage of the integration of calculus and physics in the new classroom. These examples illustrate what we consider to be the important advantages of our integrated syllabus and classroom: each subject motivates the other and each reinforces the other; understanding in physics is used to deepen understanding in math and that deeper understanding is used again to sharpen insight in physics, back and forth, again and again, throughout the semester. The physical layout of the classroom supports this process with the availability and close integration of technology and the immediate availability of suggestions and guidance from peer-leaders and faculty.

DIFFERENTIALS AND ERROR CALCULATIONS: The theory and practice of error propagation provides a great opportunity for deepening the connection between calculus and physics, as well as strengthening conceptual understanding in both fields. Establishing a clear understanding of error in measurement and error in calculated values has frustrated physics instructors, probably for centuries. Students often have a sketchy understanding of the formulas for propagated errors and for that reason often misuse them or fail to take them seriously. Similarly, calculus students struggle to gain an intuitive grasp of concepts such as the ϵ - δ definition of the limit and the concept and application of differentials. Our integrated course allows us to deepen this material substantially as described in the scenario below.

Early in the first semester students are introduced to error in measurement by timing a falling ball. In this experiment students measure position as a function of time and use this data to calculate velocity. They see that the quantity measured is rarely the quantity desired and that if we are to take error measurement seriously, we have to deal with the error in a calculated value. This experience in an experiment motivates and reinforces the calculus material in several ways. In the study of limits we will employ an analogy to error measurement in which ϵ plays the role of a desired accuracy in a calculated value and δ plays the role of the uncertainty in the measurement. The "ϵ-δ game" then becomes the fundamental question "how accurately do I have to measure x to get a desired accuracy in $y = f(x)$?" The standard example of $y = x^2$ used in every calculus book now has added meaning in the context of a measured velocity v being used to calculate a desired kinetic energy $\frac{1}{2}mv^2$ (with known mass m.) This exact measurement is done in Workshop Physics, where students measure position and time for a falling ball and demonstrate conservation of mechanical energy. From the measured quantities, potential and kinetic energy are then calculated. One expects that the sum of these will be constant along the path of the ball, but due to uncertainty they won't be equal, so the determination of these uncertainties will clearly be important. The importance of this problem (established by the physics context) and the difficulty of the ϵ-δ calculations set the stage for another standard topic in the first semester of calculus – differentials.

In typical calculus texts the application of differentials to error propagation is invariably limited to the calculation the of the error in $y = f(x)$ given some uncertainty in the measurement x. In our extension of the WP materials we address the more practical, interesting and realistic problem (raised by the experiment) of finding the allowable uncertainty in the measurement needed to achieve a desired accuracy in the calculated value. Because of the integration of the two subjects, we are able to motivate and effectively treat this case. The syllabus will be arranged so that the solution of the extended problem will be taken up concurrently in the physics and the calculus portions of the class. These interactions, made possible by the tight integration of the two subjects, reinforce the important concepts of measured and calculated errors in physics and give a hands-on feeling for differentials in calculus.

Riemann Sums and Electric Field of a Line Charge: Every beginning physics student calculates the electric field generated by a line charge. Usually an argument for the validity of the method makes reference to the concept of Riemann sum which "you all remember from calculus." But perhaps the students haven't had it in calculus yet, or perhaps it wasn't emphasized or no connection was made to a physical model of a Riemann sum. Our integrated syllabus and classroom make possible important improvements of this unfortunate (and common) situation. The physics instructor will know that her students are familiar with Riemann sums because her teaching partner will have just covered that topic the previous week. Moreover, one of the motivating examples will have been the exact line charge problem that the students are faced with in this lab. Strengthening the connection will be the use of a common software tool (Mathematica) to calculate and visualize the Riemann sums. In the calculus class, we will have investigated limits of Riemann sums both theoretically and experimentally using a small-group interactive computer lab. The existing WP materials only deal with a single Riemann Sum, a uniform charge distribution and the field at two specific points. Using Mathematica students will explore refined Riemann sums, reinforcing the important concept of the definite integral as a limit of Riemann sums, and a broad range of charge distributions and field locations.

These scenarios illustrate how the technology, Workshop Physics experimental equipment and curriculum adaptations are essential to our pedagogy. The integrated curriculum introduces a symbiosis between mathematics and physics driving understanding in each subject to higher levels as well as achieving an efficiency buying the time for this to happen. Use of Workshop Physics materials such as motion sensors, air tracks and photogates gives students a hands-on experience that increases student ownership of the material. The networked computers and the classroom common to both math and physics, allows students to share data and analyze it in a variety of contexts, promoting the ability of students to apply their knowledge across disciplines. The Mathematica notebook interface allows us to write interactive tools with a minimal technical burden on students. Using these notebooks, students have access to sophisticated tools for analyzing their data and exploring theoretical concepts.

Pedagogy

Workshop Physics is an activity-based curriculum developed at Dickinson College incorporating the outcomes of physics education research. Since 1986, the Workshop Physics project has developed a set of computer tools, experimental apparatus, and curricular materials for teaching introductory courses without lectures. This curriculum has been adopted with great success at approximately 15 colleges and 35 other institutions [13].

We will adapt these materials in two ways. First, we will overlay the PLTL model as described below. Second, the integrated course will naturally lead to extensions of the physics material as illustrated in the preceding scenarios. Another adaptation will be an increased use of technology to develop conceptual understanding and strengthen the connection between math and physics. At present, Workshop Physics uses spreadsheets for calculations which keep the mathematical ideas in the background and fail to build conceptual links to mathematics. Our students will use more sophisticated tools (Mathematica) in both the calculus and physics components of the course. By using these tools, students will be able to visualize and explore these conceptual links deepening their understanding of both subjects. But the connections run both ways. By having the Workshop Physics experimental tools easily available to calculus classes, students actively engage in experiments and bring the mathematical ideas to life.

In the peer-led team learning model, students who have done well in a class become guides and mentors to small groups of students. The calculus portion of the class will follow a typical PLTL model in which the peer-leaders are involved in those classes in which students will be doing hands-on work in their small groups. Since the physics portion of the curriculum is based on Workshop Physics, students in these sessions will always be working in their small groups and these sessions will always have peer-leader involvement.

In our implementation, the PLTL workshop is the class and the faculty member is always present. In addition, we will have regular staff meetings with the peer-leaders in which we will prepare for upcoming classes, verify peer-leader's content knowledge, and plan for adjustments and improvements. Given the emphasis the Workshop Physics curriculum makes on small-group work, it is ideally suited for use inside a PLTL model. In our case, a peer-leader works with three groups each consisting of three or four students. This differs from other implementations of the PLTL model in which peer-mentors work with single groups of six to eight students. Research [14] and our personal experience indicate that groups of three to four are optimal for this pedagogy. Further arrangements are described below in the discussion of the classroom design.

Peer-leaders will be involved in all the physics sessions and about half of the mathematics sessions. Each student group will work with one or two peer leaders throughout the semester and across both components of the course. This is important in that it strengthens the connections between the subjects and builds social relationships vital to success in the course. The experience of faculty at the University of Portland and other institutions has been that the social relationships built between students and their peer-leader and among students sharing a common peer-leader have been very valuable in terms of student satisfaction, interest, and success in the course. This effect has been pronounced for the typical case in which the peer-leaders meet with their groups once each week. We expect that it will be even stronger in our case.

Classroom Design.

The physical design of the classroom is extremely important to the success of the proposed project. The design is motivated by the educational goals of the collaborative, peer-led, integrated math and physics curriculum. The facilities, layout of the furniture and choice of equipment encourage interactions among students within groups, among the different groups, with peer mentors and instructors, and across the class as a whole. We have designed for flexibility both in the general use of the room (for an easy transition between lecture and group based work) and the activities within the room (experiments, problem-solving, computation, and discussion.) Furthermore, the flexibility allows for both guided and independent use of the facilities. We aimed for maximum efficiency in the use of the space and the integration of technology.

The floor plan includes hexagonal work stations, each fitting three to four students. At Dickinson College, Workshop Physics is taught to 24 students, working in pairs, two pairs to a station, and two computers at each station, located on the sides. The stations in our proposed classroom are similar to those used at Dickinson College for Workshop Physics, but we have made the following significant modifications to the stations and to the floor plan:

• In our classroom, three to four students will work as a group at each station. Each station will have a single computer with a large flat screen monitor. By the use of Mathematica and through the integration of physics and calculus, the computer is used much more intensively than in WP. Placing a single computer with a large screen more centrally allows better and more immediate access by the group as a whole. The large monitor is also important in allowing for a variety of activities simultaneously: data collection, analysis, theoretical calculations, as well as display of discussions across the classroom (see below.) Given the size of the monitor, flat screen technology is essential for preserving workspace and visibility.

• Each station has a cabinet in which all the workshop physics equipment for that station is stored giving students access to all of the equipment at any time. They can use it when required or when they have an original idea for an investigation during calculus class, physics class, or during open lab hours.

• To facilitate the PLTL model we arrange the stations in natural groupings of two to three stations. A peer mentor is associated with each set of stations. A whiteboard is available to each set, essentially creating a mini classroom within the larger room and encouraging interaction among groups. Across from each station is another station, providing flexibility in the interactions and a bridge between sets of stations. Stations are connected by a center strip, formed by the tops of the storage cabinets, which is available for experiments requiring longer distances (such as rolling balls over several meters.) These activities reinforce collaboration among groups.

• Careful design of a central instructor station, with a large whiteboard, computer, and video system, and the networking of all of the computers is critical to class-wide interaction. This station is visible to all students, allowing the instructor to lecture while students remain in their groups. Students share data, calculations, and discussions with other groups through the networked computers. This supports the PLTL model, in which students come to their own conclusions through discussion, and extends it by enabling group to group interactions. The instructor can also share data, observations, and calculations with the entire class or combine data collected by all of the groups. For example, data from an experiment performed at the instructor's station can be analyzed and manipulated by students. Similarly, after an example of a problem-solving technique in math or physics, the resulting Mathematica calculation can be made available to students, who then modify the problem and observe the corresponding change in the results.

Pathways Conference Workshop

The conference workshop will begin with a brief introduction to our course, including a sample syllabus and diagrams of the classroom design. Using the PLTL model, the workshop leaders will act as peer-leaders as the participants follow one or more of the thematic threads through a semester. Workshop participants will then have the opportunity to suggest and develop new thematic threads and discuss ideas for additional activities that support these threads. The opportunity for exciting, engaging examples are limitless. Here are a few more examples: Taylor Series and The Field of an Electric Dipole; Chain Rule and Gears; Limits and Friction. Through group discussion, we will explore how the use of these thematic threads supports both the integration and active engagement.

Bios

Greg Hill is an associate professor of mathematics at the University of Portland, with extensive experience in calculus education at a wide variety of institutions. With a Ph.D. in mathematics and a B.S. in physics and mathematics he has the expertise to develop and implement the changes of this project. He has developed and used a year-long set of interactive calculus labs using Mathematica for a workshop oriented calculus class. As an OCEPT fellow, he developed a set of interactive activities for abstract algebra and attended a week-long PLTL training.

Tamar More is an assistant professor of physics at the University of Portland. She has introduced tutorial style group learning into the calculus based general physics course at UP. As an OCEPT fellow, Dr. More has implemented peer-led team learning in the algebra based general physics course at UP as well. She has extensive experience with group mentoring and tutorials. At the University of Oregon, Dr. More assisted in developing and running tutorials for introductory physics.

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